Macroeconomic Theory (ECON 8106)

V.V. Chari

Fall 2016

Problem Set 2

Due Date: November 10th, 2016

Please hand in one physical copy per group and write the names of your group members on the first page. This problem set includes exercises from Stokey Lucas Prescott (SLP) and Ljungqvist Sargent (LS). Many of the questions have references. These are meant to be further reading and may not necessarily hold any answers for the question.

1. Deterministic Dynamic Programming
   Tree Cutting: SLP 5.5
   Investment with Convex Costs: SLP 5.9

2. Search & Unemployment
   SLP Exercise 10.7
   LS Exercise 6.4, 6.9, 6.11, 6.14

3. Merton (1969): Lifetime Portfolio Selection Under Uncertainty
   Consider a household that lives for a finite number of periods, T. Wealth, W_t, at the beginning of the period is allocation towards current consumption, c_t, purchases of a risky asset x_t, and a risk-free asset, y_t, according to

$$c_t + x_t + y_t \leq W_t$$

   Wealth evolves according to

$$W_{t+1} = Rx_t + R_f y_t$$

   where R denotes the gross return to the risky asset and R_f denotes the gross return to the risk-free asset. Assume that R is iid over time and drawn from a distribution $F(R)$. The household's preferences are given by

$$\sum_{t=0}^{T} \beta^t \frac{c^{1-\sigma}}{1-\sigma}$$

   .

   a. Set up the household's problem as a dynamic program.

   b. What does this model say about how a household's portfolio should change as they age? (Hint: Guess at the functional form of the value function and derive the optimal asset allocation.)

4. Angeletos Calvet (2005): Idiosyncratic Production Risk, Growth, and the Business Cycle

   Consider the problem of a consumer/entrepreneur who has access to a production technology that uses capital and labor to product a homogeneous good. Let k_t denote the amount of capital the entrepreneur has at date t, then the production function is

$$y_t = z_t f(k_t, n_t)$$

   where z_t is a random variable that is iid over time. f has constant returns to scale and n_t denotes the amount of labor *hired* by the entrepreneur. Capital fully depreciates each period. The entrepreneur can invest only in her own production and cannot rent capital to or from others. The entrepreneur can also save in one-period risk free bonds with interest rate r_t. Profits to the entrepreneur are given by

$$\pi(k, z; w) = \max_n z f(k, n) - wn$$

   The consumer/entrepreneurs problem is to maximize life-time utility, taking all prices as given,

$$\max E \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

   subject to

$$c_t + k_{t+1} + b_{t+1} = \pi(k_t, z_t; w_t) + (1 + r_{t-1}) b_t$$

   a. Write the consumer/entrepreneurs's problem recursively.

   b. Assume that all prices are constant over time. Also assume that the borrowing limit for the consumer is 0, i.e. the consumer can only save. You may still assume that this is not binding. Show that the consumer problem can be re-written as a consumer with access to a risky and safe asset for savings. Clearly lay-out this problem.

   c. Prove that the consumers portfolio composition between the safe and risky asset is constant.

5. An Inventor

   Consider the problem of an inventor. She can choose to invent a new product at a cost of b ot not. If the inventor chooses to invent, the quality of the invention, z, is randomly drawn from a distribution $F(z)$. The quality is also the per-period profit of the invention. Once a product is invented, the inventor becomes a manager and operates her business at cost w. With probability δ, this business will fail, and the manager will return to her first love, inventing. Assume the inventor is risk neutral and discounts future profits.

a. Step up the inventor's problem. What is the manager's problem in this case?

b. Derive a condition on b so that there will be any inventing.

c. Now suppose that once that once a product is invented, there is a probability θ that the business will get a one-time, permanent boost in per-period profits from z to $z + \Delta$. Write the problem of a manager who has yet to experience the productivity boost. What is the minimum z at which a manager will operate?

d. Write the inventor's problem. How does this change the condition on b?

6. Badel (2015): Racial Inequality Trap
Consider a dynastic household choosing between n neighborhoods to live in. Each period is a generation. In each period, the household consists of a parent (decision maker) and a child. The parent has human capital h, and the child has a learning ability z. The parent allocates time between working for a wage and investing in the child's future productivity. The parent purchases consumption, c, and housing services g. The price of housing in neighborhood n relative to consumption is P_n. The household budget in a given neighborhood is

$$c + P_n g \leq wh(1 - l)$$

In the following period, the child becomes the parent with human capital $h' = F(z, hl, P_n)$, where F is strictly increasing in all arguments, and a new child is born with new productivity, z'. Productivity takes discrete values $z \in \{\underline{z}, \ldots, \bar{z}\}$ and follows a markov chain, i.e. $z' \sim \pi(\cdot, z)$. Per-period utility is given by

$$u(c, g)$$

a. Write the problem of a dynastic household living in neighborhood n. Assume that prices are constant.

b. Characterize the optimal decision rules of a household living in neighborhood n. How do these decisions depend on the price of housing? On the learning ability of the child? You may make any assumptions you feel necessary to get a more succinct result.

c. Write the problem of deciding in which neighborhood to live. How does this decision depend on the learning ability of the child? You may make any assumptions you feel necessary to get a more succinct result.