Macroeconomic Theory (ECON 8106)

V.V. Chari

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Problem Set 1

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Please hand in one physical copy per group and write the names of your group members on the first page.

1. Consider the following *Edgeworth economy* with two commodities (say apples and bananas). There are two (types of) households (labeled 1 and 2). The preferences of both households are the same and are given by

$$u\left(c_a^i, c_b^i\right) = \log\left(c_a^i\right) + \beta \log\left(c_b^i\right).$$

Household 1 is endowed with w_a units of apples and 0 units of bananas, and household 2 is endowed with 0 units of apples and w_b units of bananas.

   (a) Define a competitive equilibrium.

   (b) Suppose $w_a = w_b = 1$ and $\beta = 1$. Compute the competitive equilibrium (or equilibria, if there is more than one).

   (c) What conditions do the set of Pareto optima have to satisfy?

   (d) Suppose $w_a = w_b = 1$ and $\beta = 1$, as in Part (b). What prices and transfers support the following allocation:

$$\left(c_a^1, c_b^1\right) = \left(\frac{3}{4}, \frac{3}{4}\right).$$

2. Consider the following 2-period economy; there is a single consumption good in each period. There are 2 households who have identical preferences over consumption given by

$$u\left(c_1^i, c_2^i\right) = \log\left(c_1^i\right) + \beta \log\left(c_2^i\right).$$

Household 1 has endowments given by $(w_1, 0)$, and household two has endowments given by $(0, w_2)$.

   (a) Define an *Arrow-Debreu competitive equilibrium.*

   (b) Suppose households attempt to smooth their consumption over time by borrowing and lending. Define a *sequential markets competitive equilibrium.* Show that a sequential market's equilibrium and an Arrow-Debreu equilibrium give identical allocations.

   (c) Suppose $w_1 = w_2 = 1$ and $\beta = 1$. Calculate the competitive equilibrium.

(d) Suppose households cannot borrow from or lend to each other. Try to define a competitive equilibrium in this case. Show that a policy which transfers goods from the rich to the poor in each period is *Pareto-improving*.

3. Consider the following savings problem; an infinitely-lived household has preferences over consumption at each date given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t).$$

The household has wealth given by W_0 in period 0. The budget constraint in each period is

$$c_t + W_{t+1} \leq RW_t.$$

Also, assume $c_t, W_{t+1} \geq 0$ for all $t \geq 0$.

(a) Suppose $\beta R = 1$. Set up and characterize the solution to the household's problem as a choice of sequences.

(b) Suppose $\beta R \neq 1$. Characterize the solution to the household's problem.

(c) Set up the household's problem as a dynamic program and solve Parts (a) and (b).

4. Compare two *one-sector growth economies* which are identical except that in economy 1, the production function is $A_1 k_t^{\alpha} l_t^{1-\alpha}$, and in economy 2 it is $A_2 k_t^{\alpha} l_t^{1-\alpha}$. The preferences of households are

$$\sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \varphi \log(1 - l_t)\right].$$

Compare wage rates and rental rates in the *steady states* of the two economies.

5. Consider a one-sector growth model in which $y = f(k, 1)$, in each period, and preferences are

$$\sum_{t=0}^{\infty} \beta^t u(c_t).$$

(a) Set up the social planner's problem as a dynamic program.

(b) Prove that the policy function is increasing in the capital stock.

6. Consider a two-period economy in which the representative household's preferences are $u(c_1, c_2)$. Suppose the household has an endowment of y_1 in period 1 and y_2 in period 2, and the interest rate is R. Assume the government levies a proportional tax on savings, τ, and redistributes the proceeds in a lump-sum fashion. Give sufficient and (as best as you can) necessary conditions for an increase in τ to reduce savings. Assume $u(\cdot)$ is concave and twice differentiable.

7. Consider the following savings problem; an infinitely-lived household has preferences over consumption at each date given by

$$\sum_{t=0}^{\infty} \beta^t \log(c_t).$$

The household has wealth given by W_0 in period 0. The budget constraint in each period is

$$c_t + W_{t+1} \leq RW_t.$$

Assume $c_t, W_{t+1} \geq 0$ for all $t \geq 0$. Suppose also that the return on wealth is logarithmically distributed with mean μ and variance σ.

   (a) Set up the household's problem as a sequence problem.

   (b) Characterize the solution. What can you say about consumption as a function of RW?

   (c) Try and set up the household's problem as a dynamic program. Characterize the policy function.

8. Consider the economy of Question 7, except that the household's preferences are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where

$$u(c_t) = \frac{c_t^{1-\theta}}{(1-\theta)}.$$

   (a) Set up the household's problem in sequence form.

   (b) Set up the household's problem as a dynamic program.

   (c) *(difficult)* Show that the household's asset allocation is constant if R is iid over time.

9. Consider the following neoclassical growth model with uncertainty; in each period, there is a realization of an exogenous random shock, s, that takes values in a finite set, S. Let us denote the history of shocks up to date t by $s^t = (s_0, s_1, \ldots, s_t)$, for $t = 1, 2, \ldots$. There is an infinitely-lived representative household in the economy, whose preferences over consumption allocations are represented by the following utility function:

$$E_0 \sum_{t=0}^{\infty} u\left(c_t\left(s^t\right)\right),$$

where the expectation is taken with respect to the stochastic process governing exogenous shocks, and the initial realization of s_0. Household is endowed with k_0 units of physical capital at the beginning of time, and one unit of time in each period.

There is a firm that has access to a constant returns to scale technology, denoted by $A(s) F(K, H)$, that transforms capital services and labor into a composite good. This good can then be sold to the household. $A(s)$ is a productivity shock that is a functional of the realization of s.

Suppose household sells all its endowment of capital to the firm at the beginning of time, and firm makes all the investment decisions afterwards. The resource constraint in the economy is

$$C_t + K_{t+1} = (1-\delta) K_t + A(s) F(K, H).$$

(a) Define the commodity space, consumption set, and production set for this economy.

(b) Define the Arrow-Debreu equilibrium, using the elements defined in Part (a).

(c) Suppose, instead of selling the initial endowment of capital to the firms, households keep the capital and make the investment decisions. Prove this leads to the same equilibrium allocation as in Part (b).

(d) Now, assume household holds the capital in the first period, and make investment decision for one period. After the shock in period $t = 1$ is realized, s_1, markets open once more, and household sells the capital to the firm. What can you say about the allocation that results in the equilibrium from history $s^1 = (s_0, s_1)$, onward?

10. Consider a two-period economy with two types of agents of equal measures. The endowments for all agents in period 0 are e_{10}, e_{20}. In period 1, both agents are endowed with $e_1(s)$ and $e_2(s)$, where $s \in \{s_H, s_L\}$. Denote s^t the history of shocks up to period t. The utility function for agent i is defined as

$$\frac{c_{i0}}{1-\sigma} + \beta \sum_s \mu(s) \left[\frac{c_{i1}^{1-\sigma}(s)}{1-\sigma}\right]$$

(a) Define an Arrow-Debreu Competitive Equilibrium for this economy.

(b) Suppose that

$$\begin{aligned} s_L &: \ e_1(s_L) = \underline{e} \quad e_2(s_L) = \bar{e} \\ s_H &: \ e_1(s_H) = \bar{e} \quad e_1(s_H) = \underline{e} \end{aligned}$$

Show that the consumptions in period 1 for both agents are independent of the shock s, that is $\forall i \in \{1, 2\}, c_{i1}(s) = c_i$

11. Consider a two-period endownment economy where there are M types of agents. There is one good in the first period, and N goods in the second period. Agent $i \in \{1, ..., M\}$ receives endownments given as

$$e_i^0, e_i^1(s)$$

where $s \in \{s_1, \dots, s_N\}$ a stochastic shock.

(a) Define an Arrow-Debreu Competitive Equilibrium

(b) Suppose the only markets are Arrow securities $a(s), \forall s$ that promise to provide a units of goods in terms of period 0's good at state s. Specifically,

$$a(s) = (0, \dots, a, 0, \dots 0)$$

$\forall i \in \{1, 2\}$, the sequential budget constraints are

$$\begin{aligned} c_{i0} + \sum_s q(s) a_i(s) &\leq e_0 \\ p(s) c_i(s) &\leq a_i(s) + e_i(s) \end{aligned}$$

Define a sequential markets competitive equilibrium with one-period Arrow securities.

(c) Show that an Arrow-Debreu equilibrium outcome is also an equilibrium outcome with Arrow securities.