Macroeconomic Theory (ECON 8105)

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Problem Set 4

Due Date: October 21st, 2016

Please hand in one physical copy per group and write the names of your group members on the first page.

Question 1: Ramsey's Problem & Optimal Taxation (Prelim SP2014 Q.II)

Consider an infinite horizon setting in which there is a representative consumer and a representative firm as in the standard single sector growth model. The utility function of the representative consumer is given by:

$$\sum_t \beta^t u(c_t, l_t)$$

The feasibility constraint for the firm is:

$$c_t + x_t + g_t \leq F(k_t, n_t)$$

where g_t is the amount of government goods and services produced in each period.

There is no technological change. Investment is done at household level, and the standard law of motion for capital is assumed to hold.

Suppose that government has at its disposal only on labor income and consumption taxes for financing expenditures (i.e. no capital income taxes), but can freely borrow and lend (i.e., it faces a present value budget constraint). Assume that the consumer takes $g_t, \tau_{nt}, \tau_{ct}$ as given when making decisions.

a. Set up and define a TDCE (Tax Distorted Competitive Equilibrium) for each fixed sequence $\{(g_t, \tau_{nt}, \tau_{ct})\}_{t=0}^{\infty}$.

b. Assume interior solution of the equilibrium, give the FOC's that characterize the equilibrium you defined in Part (a).

c. What is the Implementability Constraint for this economy? Compare this to what you would get if the government could choose labor and capital income taxes, but not consumption taxes.

d. If g_t is a fixed sequence and the government acts benevolently in choosing (τ_{nt}, τ_{ct}), will it be true that $\tau_{ct} \to 0$? Does $\tau_{nt} \to 0$? If not, what can you say about $\lim_{t\to\infty} \tau_{ct}$ and $\lim_{t\to\infty} \tau_{nt}$?
(Assume that in the Ramsey allocation, all quantities converge to constant levels, $c_t \to c_\infty$, etc.)

Question 2: Optimal Taxation and Human Capital (Half of Prelim SP2016 Q.I)

This problem asks you to work with the version of the neoclassical growth model with human capital and exogenous labor supply. Consider the following Planner's Problem:

$$\max \sum \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to:

$$c_t + x_{kt} + x_{ht} \leq y_t = A k_t^\alpha z_t^{1-\alpha} \tag{1}$$
$$k_{t+1} \leq (1-\delta_k)k_t + x_{kt} \tag{2}$$
$$h_{t+1} \leq (1-\delta_h)h_t + x_{ht} \tag{3}$$
$$z_t \leq n_t h_t \tag{4}$$
$$0 \leq l_t + n_t \leq 1 \tag{5}$$
$$h_0,\ k_0, \text{ given.} \tag{6}$$

Suppose that the government has a fixed sequence of expenditures that it must finance, g_t, and that it can use taxes on capital and labor income, τ_{kt} and τ_{zt}.

a. Define a competitive equilibrium for this environment.

b. What is the Ramsey Problem here for a benevolent government? In particular, carefully derive and explain the implementability constraint for this environment.

c. Assume that $\delta_k = \delta_h$. What is $\dfrac{\tau_{kt}}{\tau_{zt}}$ in this case?

d. Assume that $\delta_k = \delta_h$. What can you say about $\lim_{t\to\infty} \tau_{kt}$? What about $\lim_{t\to\infty} \tau_{zt}$?

Question 3: Stochastic Taxes (Prelim FA2015, Q.II)

Consider the Ak model, i.e. $y_t = Ak_t$, of savings and growth with full depreciation and a representative consumer with preferences given by:

$$U = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right]$$

Assume that the government taxes all income in period t at the rate $0 < \tau_t < 1$, where τ_t is any stochastic process. Assume that all revenue generated in period t is used in that period to purchase goods and services, i.e. $p_t g_t = \tau_t r_t k_t$, in all states and all dates.

a. Show that the TDCE allocation for this model solves a Planning Problem. Be sure to carefully and clearly state what this planning problem is.

b. Assume that the tax rates, τ_t, are i.i.d. Under what conditions is it true that an increase in the spread of the distribution of τ (in second order stochastic dominance sense), holding the mean of τ fixed, increases the mean growth rate of consumption for the household? Clearly state the conditions, and <u>show</u> that these conditions are true

Question 4: Stochastic Dynamic Programming

Read sections 9.1 and 9.2 in SLP and <u>solve exercise 10.1</u> by applying the assumptions and theorems of chapter 9.