Macroeconomic Theory (ECON 8105)

Larry Jones

Fall 2016

Problem Set 3

Due Date: October 11th, 2016

Please hand in one physical copy per group and write the names of your group members on the first page.

Question 1: Policy Function–Discount Factor (Prelim SP2008, Q. II.1)

Here you are asked to show that the policy function of a standard growth model is increasing in the discount factor. Let T_β be the operator of mapping the set of weakly concave, bounded, continuous functions $f : X \to \mathbb{R}$ into itself: $T_\beta : C'(X) \to C'(X)$.

$$T_\beta(v)(k) = \max_{k' \in \Gamma(k)} \{u\left(f(k) - k'\right) + \beta v(k')\}$$

Fix $\hat{\beta} > \beta$. Let $T_{\hat{\beta}}$be the operator using $\hat{\beta}$.

a. Argue that there is a unique fixed point for each operator.

   Let $v^*(\cdot;\beta)$ and $v^*(\cdot;\hat{\beta})$ be the fixed point for the operator T_β and $T_{\hat{\beta}}$ respectively. Assume that both functions are differentiable.
   Also, let $v_k(\cdot;\hat{\beta})$ be the k-th iteration using the operator $T_{\hat{\beta}}$ and $g_k(\cdot;\hat{\beta})$ be the corresponding policy function.

b. Show the optimality condition that $g^*(k;\beta)$ satisfies.

c. Now, use $v^*(\cdot;\beta)$ as the initial guess for the operator $T_{\hat{\beta}}$. Show the opimality condition that $g_1(k;\hat{\beta})$ satisfies.

d. Show that $g_1(k;\hat{\beta}) > g^*(k;\beta)$.

e. Show that $v_1'(k;\hat{\beta}) > v^{*'}(k;\beta)$. (Hint: Think of the envelope condition.)

f. Show that $g_n(k;\hat{\beta}) > g^*(k;\beta), \forall n$

Question 2: Exogenous Growth (Prelim FA2008, Q. I.1)

Consider the Planner's Problem version of the single sector growth model with labor augmenting technological change:

$$\begin{aligned} \max & \sum_{t=0}^{\infty} \beta^t u\left(c_t, l_t\right) \\ s.t. \quad & c_t + x_t \leq F(k_t, A_t n_t) \\ & k_{t+1} \leq x_t + (1-\delta) k_t \\ & l_t + n_t \leq 1 \\ & c_t, k_t, l_t, n_t \geq 0 \\ & k_0 > 0 \text{ given} \end{aligned}$$

where $A_t = \gamma^t A_0$.

a. Give a set of conditions under which the solution to this problem can be obtained by solving a stationary dynamic programming problem.

b. (Correction - 10/5/2016): Explicitly add enough conditions to your answer in (a) for you to show that $\lim_{t\to\infty} \frac{k_{t+1}}{k_t} = \gamma$, and actually show it.

Question 3: TDCE (Prelim SP2006, Q. I.1)

a. Define a general Taxed Distorted Competitive Equilibrium for the one-sector growth model with multiple households and multiple firms.

b. Show that if at an allocation and price system satisfy that:

- The feasibility constraint for each firm holds with equality
- The budget constraint for each household holds with equality
- All markets for all goods clear with equality

Then the Government Budget constraint is automatically satisfied. Make sure you clearly state any assumptions you need to make.

Question 4: Equivalent Taxes (Prelim SP2005, Q. I.3)

Consider a dynamic, representative agent, one sector economy in which the government must finance a given stream of expenditures, $\{g_t\}$, using only consumption and labor income taxes, τ_{ct} and τ_{nt}.

a. Define a Tax Distorted Competitive Equilibrium (TDCE) for this setting.

b. Show that any allocation resulting in an equilibrium of this sort can also be realized as an equilibrium in a world where the government must finance the same sequence of expenditures $\{g_t\}$ but can only use labor and capital income taxes, $\hat{\tau}_{kt}$ and $\hat{\tau}_{nt}$.

c. Is $\hat{\tau}_{nt} = \tau_{nt}$? That is, for a given sequence of government expenditures, and a given equilibrium allocation with that sequence in the two types of tax systems described above, are the two required time series of labor income taxes the same?

Question 5: Capital Taxation with Lump Sum Rebates (Prelim FA2013, Q. II)

Consider an infinite horizon, representative consumer, economy in which preferences are given by:

$$\frac{1}{1-\beta}\sum_{t=0}^{\infty}\beta^t u(c_t)$$

where c_t is consumption in period t. Each household has an initial stock of capital given by k_0 and is subject to the law of motion for capital $k_{t+1} \leq x_t$, where x_t is investment by household in period t. Assume that labor is inelastically supplied with 1 unit each period. Output is given by

$$c_t + x_t \leq F(k_t, l_t)$$

where F satisfies all of the normal conditions.

Assume that government taxes income earned on capital at rate τ in every period and that all revenue generated in this way is lump sum rebated to the households. There are no other taxes, and no other government spending.

a. Carefully define a Tax Distorted Competitive Equilibrium for this environment, given the fiscal policy.

b. Show that the allocation that results from the TDCE you defined in Part (a) can also be obtained by solving a Planning Problem with a representative consumer. Clearly and carefully lay out what this Planning Problem is, and show that its solution is the TDCE allocation from Part (a).

Now assume that τ is a tax on all income earned by the consumer each period and that government revenue is used to purchase government consumption. Thus, output is given by,

$$c_t + x_t + g_t \leq F(k_t, l_t)$$

c. Define a Tax Distorted Competitive Equilibrium for this environment, given the fiscal policy.

d. Show that the allocation that results from the TDCE you defined in Part (c) can also be obtained by solving a Planning Problem with a representative consumer. Clearly and carefully lay out what this Planning Problem is, and show that its solution is the TDCE allocation from Part (c).