Macroeconomic Theory (ECON 8105)

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Problem Set 1

Due Date: September 16th, 2016 @ 6PM

Form a group of 4/5 people. Please identify the members of your group to the TA by Thursday, September 8th. Each group has to hand in one physical copy of the homework answers. Please include the names of all group members on the cover page.

Question 1

What are the time series properties of the following over the last 50 years for the United States (what are they on average, how much do they fluctuate, are their noticeable trends, etc.) Your report should contain graphs as well as a brief description of the data.

 a. Consumption as a fraction of GNP.

 b. Investment as a fraction of GNP.

 c. Government spending as a fraction of GNP.

 d. Payments to labor as a share of GNP.

 e. Payments to capital as a share of GNP.

 f. The capital-to-output ratio.

 g. The growth rate of output per capita.

 h. The growth rate of consumption per capita.

 i. The rate of return of capital.

Question 2

Detrend the following series for the United States. Report the trends and residuals as well as their statistical properties.

 a. Output per capita.

 b. Consumption per capita.

 c. Investment per capita.

 d. Hours worked per capita.

Question 3

In the first recitation, we covered a pure exchange economy model with 2 infinitely lived consumers. Consider the same model with n infinitely lived consumers. Let

$$U^i(\underline{c}^i) = \sum_{t=0}^{\infty} \beta^t u^i(c_t^i)$$

$$u^i(c^i) = \frac{(c^i)^{1-\sigma} - 1}{1 - \sigma}$$

where $0 < \sigma$, $0 < \beta < 1$.

a. Show that if $\sigma \to 1$, then $u^i(c^i) \to log(c^i)$.

b. Define the Arrow-Debreu equilibrium for this economy.

c. Does u^i satisfy the Inada conditions? Is u^i concave?

d. Write down the first-order conditions (FOCs) for the households maximization problem.

e. Show that the above FOCs, the resource contraint, and the transversality condition are necessary and sufficient to characterize the equilibrium allocations.

Question 4

Consider an economy two infinitely lived consumers, each of whom has the same utility function $\sum_{t=0}^{\infty} \beta^t log(c_t^i)$, where $0 < \beta < 1$. Consumers' endowments are

$$\left(w_0^1, w_1^1, w_2^1, w_3^1, \ldots\right) = (4, 2, 4, 2, \ldots)$$
$$\left(w_0^2, w_1^2, w_2^2, w_3^2, \ldots\right) = (2, 4, 2, 4, \ldots)$$

a. Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, etc. Define Arrow-Debreu equilibrium.

b. Calculate the (unique) Arrow-Debreu equilibrium of this economy.

c. Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, etc. Define a sequential markets equilibrium.

d. Calculate the unique sequential markets equilibrium of the economy.

e. Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium.

f. Define a Pareto efficient allocation for this economy. Prove that the allocations in part a and b are Pareto efficient.

g. Calculate Pareto efficient allocation by maximizing a weighted sum of utility $\alpha^1 U^1 + \alpha^2 U^2$. This is called the Negishi characterization.

Question 5

Consider an economy with a representative, infinitely lived consumer who has the utility function $\sum_{t=0}^{\infty} \beta^t \log c_t$, where $0 < \beta < 1$. The consumer owns one unit of labor in each period and $\bar{k}_0$ units of capital in period 0. The depreciation rate on capital is δ. The feasible consumption/investment plans satisfy

$$c_t + k_{t+1} - (1 - \delta)k_t \leq F(k_t, l_t)$$

where $F(k_t, l_t) = \theta k_t^\alpha l_t^{1-\alpha}$. Note that F has constant returns to scale (CRS).

a. Suppose that the consumer borrows b_{t+1} bonds in period t to be paid off in period $t + 1$. The consumer's initial endownment of bonds is $\bar{b}_0 = 0$, the wage rate in period t is w_t, the rental rate on capital is r_t^k, and the interest rate on bonds is r_t^b. Write down the consumer's utility maximization problem in a sequential markets economy. Explain why you need to include a constraint to rule out Ponzi schemes.

b. Write down the Euler conditions and the transversality conditions for this problem.

c. Define a sequential markets equilibrium with borrowing and lending for this economy.

d. Derive a condition relating the rental rate of capital, interest rate on bonds, and depreciation rate. What is the tuition behind this condition?

e. Suppose that consumer can buy new capital in each period and rent capital services to the firm (as defined in recitation). Define the Arrow-Debreu equilibrium for this economy.

f. Suppose that the consumer sells his endownment of capital to the firm in period 0. Thereafter, firm invests in its own capital stock. Define the Arrow-Debeu equilibrium for this economy.

g. Carefully state the theorems that relate the equilibrium allocations in part 3 and 5. Be sure to specify the relationships between the objects.

h. Carefully state the theorems that relate the equilibrium allocations in part 5 and 6. Be sure to specify the relationships between the objects.

Question 6 Suppose that instead of an infinitely-lived firm, a separate firm exists in each period, and each of these firms makes input and output decisions only for one period. The consumers supply labor and capital to the firm in each period. Show that an equilibrium in this environment is the same as an equilibrium the environment from Question 5. Is the converse true? Show it.

Question 7 The following questions deal with a 2 sector economy with firms producing consumption goods indexed by $j_c = 1, 2, \ldots, J_c$ and firms producing investment goods indexed by $j_x = 1, 2, \ldots, J_x$. Assume CRS technology for each firm in each sector. There are n consumers indexed by $i = 1, 2, \ldots, n$ and preferences $U^i(\underline{c}, \underline{l}) = \sum_{t=0}^{\infty} \beta^t u^i(c_t^i, l_t^i)$, where $0 < \beta < 1$.

  a. Define an Arrow-Debreu equilibrium.

  b. Write down the necessary first-order conditions that a solution to each consumer's problem must satisfy.

  c. Suppose the solution to the consumer's problem is interior, i.e. all quantities at all times are strictly positive. Derive a condition relating the price of investment goods and the rental rate of capital. State briefly in words what this means.

  d. Prove that, in equilibrium, consumers cannot make positive profits by investing. Hint: Show that in equilibirum, the constraints of the consumer's problem can be re-written so that only the initial endownment of capital enters the budget constraint.

  e. Write down the necessary first-order conditions that a solution to each firm's problem must satisfy. Derive a condition relating the price of consumption goods and the price of investment goods.

  f. Prove that, in equilibrium, a firm with CRS production technology (i.e. the production function is homogeneous of degree 1) will have zero profit.

  g. Define a sequential markets equilibrium.

  h. Consider any Arrow-Debreu equilibrium. Construct prices, interest rates, and borrowing/lending amounts so that the allocation of the Arrow-Equilibrium is part of the allocation of a sequential market equilibrium at those prices.

  i. Consider any sequential markets equilibrium. Construct prices for the Arrow-Debreu environment of the economy so that the allocation of the sequential markets equilibrium (without borrowing-lending holdings) is the allocation of an Arrow-Debreu equilibrium at those prices.